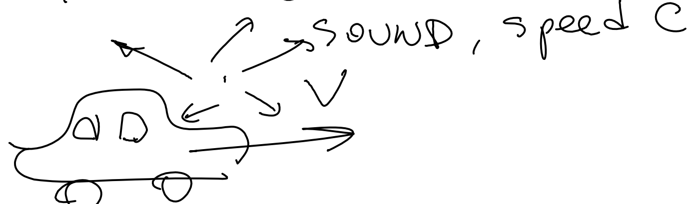
PHYS 100C, Lecture 15

Friday, May 22, 2009
9:39 PM

Michelson-Morley experiment:

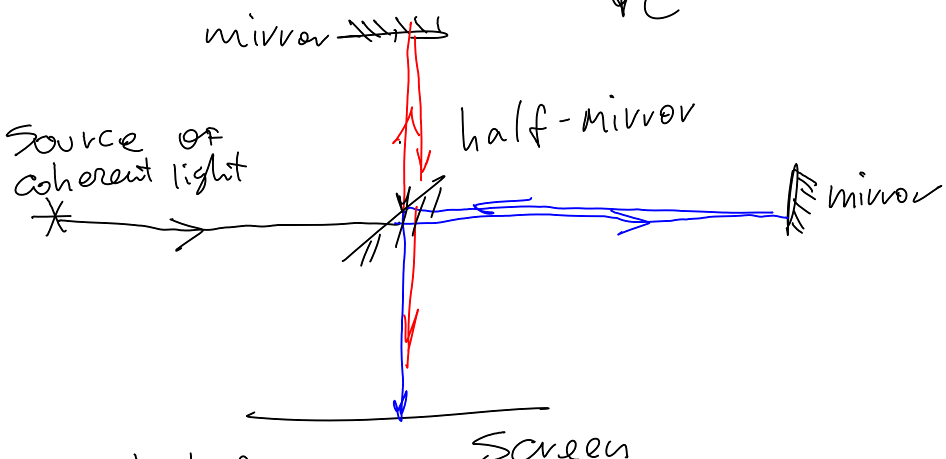
Idea: measure Earth's velocity with respect to ether.

Analogue: honk while driving a convertible:

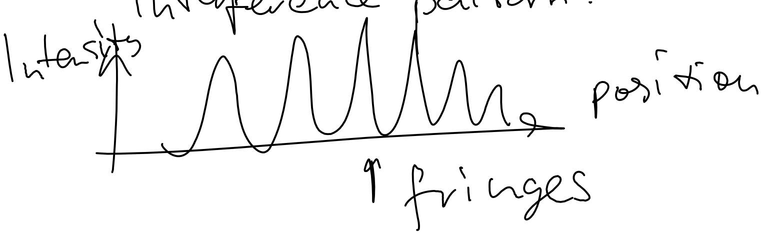


Sound relative to car: $c - v$ →

$c + v$ ←   ↓c



Interference pattern:

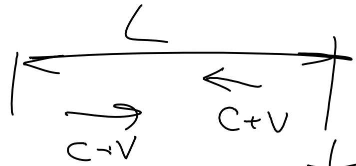


If PATH LENGTH DIFFERENCE

(PLD) BETWEEN RED & BLUE
is $m \cdot \lambda$ ($m = 0, 1, 2 \dots$ integer)
$\Rightarrow$ bright spot (constructive)

IF $PLD = m\lambda + \lambda/2$
$\Rightarrow$ dark spot
(destructive interference)

IF theory of ether is correct



time traveled:

$$t = \frac{L}{c-v} + \frac{L}{c+v} =$$

$$= \frac{2L \cdot c}{c^2 - v^2} \simeq \frac{2L}{c}\left(1 - \frac{v^2}{c^2}\right)$$

IF one arm is $\parallel$ to earth motion, another $\perp$, rotating them by 90° will change PLD by $4L \cdot \frac{v^2}{c^2}$
(one gets longer by $2L \cdot v^2/c^2$, another shorter by the same amount).

Earth orbital speed is $\approx c \cdot 10^{-4}$
FOR $L = 1\,M$, $PLD \approx 4 \cdot 10^{-8}\,M$
OR 40 nm, $1/15$th of λ_{red}.

By doing multipass approach total length of pass $2L$ was ~ 11 m in Michelson-Morley exp.
(~ 0.4 fringe width shift was expected).

Observed: zero, with precision $1/20 - 1/40$ of expected shift.

Ether theory is wrong!

* "Proper" speed

relativistic velocities add in awkward manner:

$$\tilde{u} = \frac{u - v}{1 - \frac{uv}{c^2}} \quad \text{(not linear)}$$

Introduce "proper" velocity:

$$\eta = \frac{\partial l}{\partial \tau} \begin{array}{l} \leftarrow \text{distance in lab frame} \\ \leftarrow \text{time in moving frame} \end{array}$$

$$\eta = \gamma u$$

then $\eta^0 = \gamma \cdot \frac{\partial x^0}{\partial \tau} = c \frac{\partial t}{\partial \tau} = c\gamma$

$$\eta = \begin{pmatrix} c\gamma \\ u\gamma \\ 0 \\ 0 \end{pmatrix}$$

and Lorentz transform.
(just like for $x^0, x^1, \ldots$):

$$\tilde{\eta}^0 = \gamma(\eta^0 - \beta\eta^1)$$
$$\tilde{\eta}^1 = \gamma(\eta^1 - \beta\eta^0)$$
$$\tilde{\eta}^2 = \eta^2$$
$$\tilde{\eta}^3 = \eta^3$$

Introduce "proper" momentum:

$$p = m \cdot \eta = \gamma \cdot m u$$

$$p^0 = m \cdot c\gamma \qquad (\Leftarrow \text{weird})$$

Einstein introduced it as relativistic energy E:

$$E = p^0 \cdot c = \frac{mc^2}{\sqrt{1 - v^2/c^2}} = \gamma \cdot mc^2$$

FOR $v = 0$

$$E = mc^2$$

(ANY OF YOU seen this eq. BEFORE? IT'S RATHER OBSCURE!)

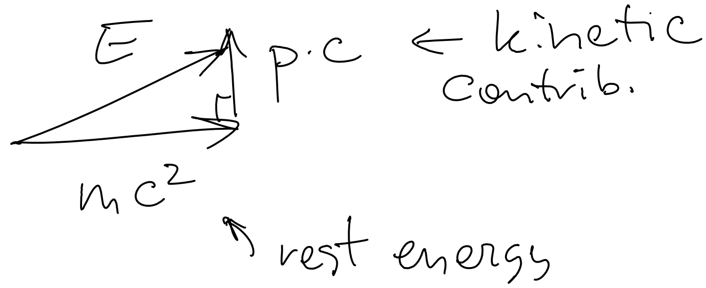
[illegible]

$$(p^\mu)^2 = -(p^0)^2 + p^2 = -\gamma^2 m^2 c^2 + \gamma^2 m^2 v^2 =$$
$$= \gamma^2 m^2 c^2 \left(\frac{v^2}{c^2} - 1\right) = m^2 c^2 \leftarrow \text{invariant}$$

↑ 4-vector

OR: (since $p^0 = \frac{E}{c}$)

$$E^2 - p^2 c^2 = m^2 c^4$$



For small v,

$$\Delta E = E_{kin} = \frac{1}{2} m v^2 + \ldots$$

(assuming rest mass does <u>NOT</u> change – no nuclear reactions etc.)

Also, if $m = 0$,

$$E^2 = p^2 c^2 \quad \text{or} \quad E = pc$$

photons: $E = h\nu$

$$p = \frac{h\nu}{c}$$